

# **Tactical and Strategic Missile Guidance**

## **Third Edition**

**Paul Zarchan**

Charles Stark Draper Laboratory, Inc.  
Cambridge, Massachusetts

**Volume 176**

**PROGRESS IN  
ASTRONAUTICS AND AERONAUTICS**

**A Volume in the AIAA Tactical Missile Series**

**Paul Zarchan, Editor-in-Chief**

Charles Stark Draper Laboratory, Inc.  
Cambridge, Massachusetts

Published by the  
American Institute of Aeronautics and Astronautics, Inc.  
1801 Alexander Bell Drive, Reston, Virginia 20191-4344

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